The ‘4D-Earth-Swarm’ project:
rapid geomagnetic field changes from Swarm

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Chapter 1

Review of the state of the art in interannual core dynamics

1.1 Context of the 4D-Earth-Swarm activities

Our activities are broadly characterised by one scientific question, namely the physical modeling of rapid secular variation (SV, or rate of change of the magnetic field) changes. These are inter-annual changes with time scales of two years to several decades. The question will be tackled using several angles of investigation, including:

- the modeling of geomagnetic data by means of reduced stochastic models of the core surface dynamics, based on satellite observations through (stochastic) data assimilation algorithms;
- the physical modeling of such SV changes through reduced quasi-geostrophic (QG) models that describe the dynamics of axially invariant motions in the core in the presence of magnetic field;
- the comparison of SV changes observed through satellite (Swarm and others) data with outputs from three-dimensional computations.

These are further described in the sections below.

1.2 Background

Swarm data hold the prospect of illuminating interior properties of the core, such as the strength and distribution of magnetic fields and, potentially, the strength of buoyancy forces. The observed spatio-temporal changes can be related to a model of the electrically conducting cores interior dynamics, provided that a predictive dynamical model of those dynamics is available. However, only in very special circumstances is such a deterministic model already available. It is the case for torsional oscillations (namely the oscillations of cylinders of fluid coaxial with the rotation axis, where the restoring force is entirely magnetic), used by Gillet et al (2010) to determine one property of the interior magnetic field from interannual changes in the fluid flow over the last few decades. In no other case is a dynamical model available for the study of the rapid (i.e. decadal and shorter) geomagnetic field changes.

The exploration of suitable strategies for the creation of a model applicable to Swarm data is one of the aims of the present proposal. The accepted state of the art for combining observations with a dynamical model is termed data assimilation (DA). At present there are two flavours of DA which are available to the geomagnetic community: probabilistic (here sequential) assimilation (SDA) and variational assimilation (VDA). The sequential approach in the context of primitive magneto-hydro-dynamic equations has been pioneered by A. Fournier & J. Aubert and colleagues at IPGP and W. Kuang & A. Tangborn at NASA. More and more groups are adopting this approach, including groups in Germany and Japan.

Recently, SDA was also considered to tackle questions posed by satellite observations by means of two pragmatic approaches: either through no-cast re-analyses (i.e. no time-stepping of the deterministic model) using three-dimensional geodynamo model cross-covariances (Aubert, 2015), or by considering instead a stochastic forecast model anchored to geodynamo spatial covariances and compatible with the occurrence of geomagnetic jerks (Barrois et al, 2017).

The variational approach has been applied to simplified problems by Li et al (2014). In principle, the mechanics of this approach are in hand, but there is a need to develop a suitable model to which this approach could be applied that does not suffer from the effects of overly-large viscosity. The idea for a variational approach was also set out in Canet et al (2009) and applied to the problem of torsional oscillations.

A number of potential avenues are open for the development of a new dynamical model. We believe that there are close parallels with similar problems in oceanography, whose community has worked for many years to develop models in which the effect of viscosity is not overbearing.

We mention promising avenues: Canet et al (2009) and Labbé et al (2015) have developed a QG model of core dynamics that holds the promise of development into a suitable dynamical model for assimilation. While most of the terms in the Navier-Stokes equation can be elegantly handled by these approaches, neither of the models are able to properly treat the magnetic terms in a rigorous manner. This family of approaches will be stepping-off points in our quest to develop a suitable dynamical core for assimilation.

In the following sections we discuss the pertinent observations and techniques that have been developed by the community, what they tell us, and what is the state of play.

1.3 Origin and observability of interannual motions observed

A consensus view of the 4D Earth team is that it is a regrettable situation that the 6 year torsional oscillations have only been observed by one team, namely the original discoverers (Gillet et al, 2010). Despite the strong evidence from the predicted length-of-day (LOD) changes that correspond well to the observed changes (Gillet et al, 2015), there is a need for an independent corroboration of these motions (even though the above observation has been confirmed with several rather distinct algorithms, see Gillet et al, 2019). This was never proposed as part of the WPs of the present proposal, but, considering the importance of the observation for core dynamics, it is to be hoped that a scientific team will take up the challenge. A key ingredient in the isolation of torsional oscillations at interannual periods by the Grenoble group is the inclusion of unmodelled SV sources associated with time-correlated subgrid processes. We believe any attempt at reproducing this result should involve this mechanism, in order to avoid either losing information by under-fitting SV data, or generating severely biased core flow models by over-fitting them.

The strongest repeating signal in LOD series is at 6 years (Abarca Del Rio et al, 2000; Chao et al, 2014; Holme and De Viron, 2013). Filtered around this period, core flow models inverted from SV models show an outward propagation of zonal motions. When interpreted as torsional Alfvén waves (Braginsky, 1970), the recovered wave form raises several geophysical issues. First the absence of noticeable reflexion at the equator may be interpreted in term of a relatively weak conductance of the lower mantle (of the order of $3\,10^7$ S), in a scenario where the core-mantle coupling is operated through an electro-magnetic stress (Schaeffer and Jault, 2016). However, there is still the possibility for a topographic torque to be responsible for the associated LOD changes (see §1.9).

Second, the propagation from the inner core (at least during the 1960-70's) has been first interpreted through a torque involving the inner core. This latter may be associated with Lorentz forces on the vicinity of the tangent cylinder (Teed et al, 2015), as it is the case in dynamo simulations (Schaeffer et al, 2017). Alternatively, it may involve a gravitational coupling between the inner core and the mantle (Mound and Buffett, 2006), although this scenario itself is debated (Davies et al, 2014; Chao, 2017). The possibility of an excitation induced by magnetospheric field changes has been proposed (Legaut, 2005), but there may not be enough energy there to excite torsional Alfvén waves (by definition equi-partitioned in kinetic and magnetic energies) with the observed amplitude. Finally, one cannot rule out the possibility of a forcing spread throughout the fluid core, as we have only access to the gravest of the torsional modes (Gillet et al, 2017). The question whether the better spatio-temporal resolution offered by Swarm data will give or not access to higher harmonics is open.

Regardless, one should keep in mind that the above zonal flows only represent a tiny contribution to interannual motions (Gillet et al, 2015; Kloss and Finlay, 2019), and that we still miss a conclusive interpretation of the more energetic non-zonal motions (see also §1.6). We also stress the limited access to interannual field changes, which are constrained by observations only for the largest length-scales in geomagnetic field models (Gillet, 2019).

1.4 Basic mechanisms for core-mantle coupling, and settled questions

Core-mantle coupling plays an important part in the time evolution of the LOD, with periods above 2 years, and in the dissipation of the annual retrograde nutation of the Earth's rotation axis. Changes of axial core angular momentum are estimated from models of the geostrophic motions in the Earth's fluid core and changes of axial mantle angular momentum are directly inferred from LOD observations. There is reasonable evidence that variations in the core and mantle axial angular momentum compensate although uncertainties remain significant (Gillet et al, 2015; Bärenzung et al, 2018). Curiously, the agreement appears less good during the satellite era, from $\approx$ 2003 onward (Gillet et al, 2019). The core-mantle coupling mechanism responsible for the exchanges of angular momentum between the fluid core and the solid mantle is still debated.

The most widely studied coupling mechanisms between core and mantle are viscous, gravitational, topographic and electromagnetic (EM). Unfortunately they all depend on poorly known properties of the lowermost mantle and core, respectively the effective core viscosity, geometry of the gravity equipotential surface next to the core-mantle boundary, topography of the core-mantle interface and the electrical conductivity of the lowermost mantle (Roberts and Aurnou, 2011). Of these, we briefly review EM coupling below as this is most relevant to the 4D-Earth-Swarm proposal.

Studies of EM sounding from Earth's surface based on external magnetic field fluctuations have poor sensitivity to the lowermost mantle, although typical values are 10 S/m (Constable, 2007). Yet due to inhomogeneities on the core-mantle boundary, these values may not be indicative of the conductivity at the interface itself. The difficulty in determining conductivity is further compounded by the fact that EM coupling mechanisms generally depend on conductance, the integrated conductivity over a layer (whose thickness is unknown), rather than the conductivity itself.

Independently, through respectively models of EM coupling and consideration of nutations, both Holme (1998) and Buffett et al (2002) propose a conductance of 10^8 S. One possibility is this is caused by a thin layer (of about 200 m) of material with the same conductivity of the core. The occurrence of a solid metallic layer at the lowermost mantle pressure and temperature is problematic and the mechanism of nutation dissipation remains an open question (Buffett, 2010). Even if the conducting materials are distributed over a thicker region, it is difficult to avoid a layer of relatively conductive material on the core-mantle boundary (CMB) interface as on treating the majority of the lowermost mantle as a single layer of depth 1000 km would yield a conductance of 10^7 S, inconsistent with other estimates. However, this reasoning does not hold if other mechanisms participate in the coupling of the core with the mantle as the required EM torque would be lower.

Another line of investigation comes from jointly considering dynamics in the core interior and interactions with the mantle. For example, torsional waves, which propagate as Alfv́en waves in the Earth's core, have periods about 6 years. Their reflection upon arrival at the core equator depends on the electrical conductance of the lowermost mantle (Schaeffer and Jault, 2016). They are completely absorbed for a mantle conductance of $1.6 \pm 0.3 \times 10^8$ S (error bar arising from uncertainties on the intensity of the radial magnetic field at the core equator). The apparently weak reflection of the waves leads to estimates of total mantle conductance in the range $3 \times 10^7 - -3 \times 10^8$S. All the above estimates offer consistent values of the conductance of about 10^8 S, although the actual electrical conductivity at the CMB is not well constrained.

Most dynamo simulations do not include magnetic core-mantle coupling. The recent geodynamo study of Aubert and Finlay (2019) dedicated to the rapid dynamics of the Earth's core however does include a thin mantle layer of conductance of about 2×10^8 S, *i.e.* comparable to the above values.

1.5 How good is the quasi-geostrophic assumption?

In rotating fluid dynamics, a geostrophic equilibrium is a balance between Coriolis and pressure forces. The only truly geostrophic motions in a rotating spherical shell are zonal (axisymmetric azimuthal) flows with axial invariance. All other flows (including convective poloidal motions) rather obey a degenerate form of geostrophy which is known as quasi-geostrophy (QG) at the condition that the first-order forces driving those flows are much weaker than the leading-order pressure and Coriolis forces. Because of the Taylor-Proudman theorem, QG flows generally acquire a quasi-invariant structure along the rotation axis when the first-order forces are sufficiently weak, leading to the possibility to formulate their dynamics in framework of reduced dimensionality (e.g. Gillet and Jones, 2006; Labbé et al, 2015; Calkins, 2018). This in turn enables important computer cost savings when performing numerical simulations, and the possibility to reach strongly turbulent regime that are appropriate for planetary cores (e.g. Gastine, 2019). QG has proven to be an efficient way to describe rapidly-rotating thermal convection (e.g. Gillet and Jones, 2006). In this non-magnetic case, the results compare favourably with three-dimensional reference models and laboratory experiments, particularly concerning the scaling behaviour in turbulent conditions

(Aubert et al, 2003; Gastine et al, 2016; Guervilly et al, 2019) because the first-order buoyancy and inertial forces remain sufficiently subdominant relative to the leading-order QG equilibrium.

Systematic surveys of three-dimensional numerical dynamos (Schwaiger et al, 2019) performed over a wide range of the accessible parameter space (including conditions approaching those of the Earth's core, Aubert et al, 2017) have confirmed the existence of a leading-order QG equilibrium even in the presence of a self-sustained magnetic field. Magnetostrophy, where the magnetic force can reach leading order and balance the Coriolis and pressure forces, is never observed at system scale (because the system needs buoyant driving) and is usually deferred to scales of about 100 km, but can approach larger scales in selected regions of the parameter space where the convective forcing is low (Dormy, 2016; Schwaiger et al, 2019). In all simulations, the occurrence of local magnetostrophy corresponds to the Lorentz force being reduced to a magnetic pressure gradient without a dynamical influence, meaning that from a dynamical standpoint QG in fact holds at all scales. In the numerical dynamos, the first-order force balance coming after QG is between the Lorentz, buoyancy forces and the ageostrophic part of the Coriolis force. This balance is known as the MAC balance and the total (leading plus first) order force balance is referred to as the QG-MAC balance. The first-order MAC balance is additionally scale-dependent. At scales larger than about 1000 km the first-order balance is mainly of thermal wind nature (balance between the ageostrophic Coriolis and buoyancy forces), with the magnetic force being subdominant. The scale-dependence of the force balance can also be viewed as a frequency-domain dependence, where time scales longer than the secular overturn are mainly governed by thermal wind dynamics and the role of magnetic forces is deferred to faster, interannual to decadal dynamics (Schaeffer et al, 2017; Aubert, 2018). This corresponds to a minimisation of the interaction between the magnetic field and the flow if sufficient time is allowed for the moderating effects of Lenz law to take place.

Unlike non-magnetic rotating convective systems, numerical dynamos frequently feature a first-order MAC balance less than an order of magnitude below the leading-order QG equilibrium (Schwaiger et al, 2019). Because of this, the slowly-varying (secular) flows can show departures from QG and axial invariance, and need to be removed in order to exhibit structures closer to QG that can be modelled as such in two space dimensions. Of particular importance are magneto-inertial waves such as interannual Alfvén waves, which have been observed in numerical simulations at the axisymmetric (e.g. Schaeffer et al, 2017) and non-axisymmetric (Aubert, 2018) levels. These latter QG, axially invariant, non-axisymmetric waves have been related to the occurrence of geomagnetic jerks (Aubert and Finlay, 2019), underlining the relevance of a QG framework to describe the geomagnetic signal at interannual time scales. The main difficulty is that the waves ride on a three-dimensional, strongly heterogeneous, slowly evolving thermal and magnetic background state that cannot readily be described within a QG framework, as stated above. This rationalises the general difficulty encountered by the community in obtaining working self-sustained dynamos that are purely QG, while more success has been obtained by studies where QG flows are produced within an imposed, rather than self-sustained, magnetic field (e.g. Labbé et al, 2015; More and Dumberry, 2017)

In summary, QG is a numerically efficient and easy to implement approximation, that has potential to describe some of the interannual core dynamics. The insight from current three-dimensional numerical dynamos however suggests that in the presence of a self-sustained magnetic field, a QG description of core dynamics most likely fails to describe the slowly-varying, buoyancy-driven secular evolution of the core that generates the field. The way to progress may therefore consist in an estimation of a three-dimensional background state (thermal, magnetic, kinematic) for the core at present (during the Swarm era), over which a QG model may be built to describe the rapidly-evolving part of the geomagnetic signal as an induced perturbation of an imposed background field.

1.6 Stochastic models anchored to geodynamo spatial covariances

There is currently a debate concerning the existence of a specific signal at 6 yr in the magnetic field. On the one hand, secular acceleration (SA) pulses, or maxima in the SA norm, seem to occur every 3 yrs (e.g. Finlay et al, 2016). This may either result from a SV signal specific to the 6 yr period (e.g. Soloviev et al, 2017), or be the consequence of the filtering in space and time when building global models (Gillet, 2019). The existence of jerks events isolated in time is particularly intriguing since we are aware of no other geophysical system displaying such a behavior. Alternatively, SA pulses could result from the spectral index $\alpha \simeq -2$ found for the temporal spectrum of SV Gauss coefficients at decadal to annual time-scales, $S(f) \propto f^{\alpha}$ (Lesur et al, 2017).

In this context, one expects the SA temporal spectrum to be flat from annual to decadal periods. The framework of stochastic processes has thus been considered for the integration of magnetic field evolution into SDA tools that only model the core surface dynamics, still incorporating geodynamo constraints by means of spatial and temporal cross-covariances (Barrois et al, 2017; Gillet et al, 2019). This approach presents the advantage of reducing considerably the dimension of the model state w.r.t. geodynamo driven DA algorithms (e.g. Fournier et al, 2013; Sanchez et al, 2019). It also extends down to annual periods the range of frequencies where the -2 spectral index operates (extreme 3D simulations, once scaled to geophysical units, lose this property at about 30 yr periods (Aubert, 2018), i.e. outside the very period range of interest for this proposal). The main current limitation of stochastic models is their inability to directly relate the observed SV changes to dynamical properties deep in the fluid core (though its products can be used as a constraint for subsequent dynamical analysis).

1.7 The role of buoyancy and of Lorentz forces

In geodynamo simulations run at high rotation rates (Schaeffer et al, 2017; Aubert et al, 2017), Lorentz forces appear to play a relatively minor role at large length-scales, and this despite a large magnetic field intensity (as measured by Elsasser numbers of order unity). Magnetic and velocity fields seem to self-organize so as to minimize induction as much as possible. The magnetostrophic equilibrium (where both Lorentz and Coriolis forces balance the pressure gradient) is thus expelled towards small length-scales (Aurnou and King, 2017), while geostrophy applies at the largest length-scale, at which departures from geostrophy are buoyancy-driven. If this scenario applies in the Earth's core, models based on magnetostrophy (see Hardy et al, 2018) might miss a crucial ingredient in order to model decadal field changes – one may think here in particular of QG models based on quadratic quantities of the magnetic field (see Jault and Finlay, 2015).

Numerical dynamos along the path are nevertheless run at parameters different from Earth-like, involving parameterizations of some nonlinear subgrid processes (Aubert et al, 2017). With lower values of the magnetic Prandtl number P_m (ratio of viscous to magnetic diffusivities), the larger magnetic diffusion may tend to enlarge the range of wave-numbers where magnetostrophy prevails. This issue is particularly important on the vicinity of the tangent cylinder. In this singular area of the core, simulations show intense magnetic fields in link with strong polar vortices (Schaeffer et al, 2017)

1.8 The prospects and applicability of the quasi-geostrophic hypothesis

We have seen in previous sections that the idea of quasi-geostrophy is attractive, as it captures much of the required physics. In the hydrodynamical case, where there are no magnetic forces, the approach can be readily used to model buoyancy-driven flows, to great effect (Guervilly et al, 2019). Presently what is missing is a theory that is able to handle the Lorentz forces that arise in the presence of magnetic fields.

A first attempt at the problem was made by Canet et al (2009). The approach to project the dynamical equations onto the equatorial plane involves an integration along the rotation axis from the lower to the upper boundary.

This integration leads to boundary terms that are, unlike all other quantities, controlled by values of electrical currents that are not describable on the equatorial plane. It was initially envisaged that these boundary terms would be much smaller than the volumetrically averaged terms and thus could be neglected (Canet et al, 2009). Subsequent work by Maffei (2016), amongst others, showed the difficulties that this leads to: when one considers the normal mode problem of small oscillations around a background state, one finds that the surface terms are non-negligible, particularly close to the equator. This leads to an incorrectly-posed eigenvalue problem.

Recognising this issue, Labbé et al (2015) pioneered a new approach. They showed that if the magnetic field could be written in the same form as the QG velocity field, then the projection of all quantities onto the equatorial plane could be achieved. This is a great step forward. It comes at a price however. The system treated means that the field lines of the magnetic field close within the fluid, and no field emanates from the core. In some ways this is similar to the treatment of Canet et al (2009). More worrying is the likelihood that a magnetic field in the core can really be represented in this QG form. The QG form for the velocity field is well motivated, relying, as it does, on the underpinnings provided by the Proudman-Taylor theorem, which leads naturally to first-order geostrophy. There is no such theorem that suggest that the QG form can be used for the magnetic field. Thus one must wonder to what extent the results will depend on this assumption.

To summarise, there is no presently acceptable magnetohydrodynamical QG formulation, and it remains a challenge for the future to develop one. The attractiveness of the approach, if a self-consistent one can be found, lies in its use for the purposes of data assimilation.

The data assimilation problem is the following. One has high quality maps of the magnetic field at the core-mantle boundary for the last decades and centuries that have been created from measurements taken at the Earth's surface and above. These will subsequently be termed *observations*, despite the fact that the maps are actually derived quantities. The quest is to find a dynamical model of motions in the core (and their time variations) that can account for the observations. The problem requires a *dynamical core*, namely a version of the fluid mechanics in the core. With these two ingredients, the matching process can begin. The outcome of the matching process is twofold. In principle one can deduce properties of the core such as the time-dependent buoyancy field and the interior magnetic field strength and geometry. These quantities are such that they lead to a dynamical evolution in time of core quantities, such that the observations are honoured. But in addition, the time-evolution can be followed forwards beyond the time window of the observations, into a prediction. This comes naturally, for free.

The attractiveness of the QG approach as a version of the fluid mechanics is twofold. Firstly it can operate in regimes that three dimensional dynamo models cannot reach. In particular, it is able to reduce the effects of viscosity to levels that are close to those expected within the core (Guervilly et al, 2019). More important considerations, however, are probably associated with the inverse problem that is being solved. Quite likely it is only possible to recover some forms of ”lumped parameters”, rather than full 3-D information. Thus one may have to be satisfied with field strengths and geometries reduced by averaging, rather than full recovery of 3-D toroidal and poloidal magnetic fields. Put simply, 2-D observations in time (observations on the core-mantle boundary) are unlikely to be able to recover 3-D fields. These 2D fields may well be able to recover 2-D fields as a function of time. Thus the pure counting problem argues in favour of a theory like quasi-geostrophy. The problem was highlighted by Li et al (2014).

It should be said that there has been considerable success by using 3D dynamo models as dynamical cores for Ensemble Kalman filter schemes. However, these calculations have not been able to constrain the interior buoyancy and magnetic fields in the core.

It is our hope that the present 4DEarth activity might lead to further insights and experiences that will lay the path for future data assimilation activities.

1.9 Topographic torques on a non-spherical core

So far there is no certainty in the mechanism that transfers core’s angular momentum to the solid mantle. Proposed mechanisms include electromagnetic coupling via electrically conducting lower mantle (see section 1.4 for more details), gravitational coupling via a gravitational torque between a deformed inner core and the mantle (Buffett, 1996a,b), or topographic coupling through a non-axisymmetric CMB.

For a spherical CMB the pressure torque on the mantle by any flow in the core vanishes exactly by definition. More precisely, for any CMB symmetric about the rotation axis, no changes in the LOD may be explained by the pressure torque. Investigating core flows in non-axisymmetric domains is challenging and has been limited to a few studies up until today (e.g. Kuang and Chao, 2001; Jault and Finlay, 2015; Vidal et al, 2019).

Torsional waves, with periods on the scale of a few years, have been proposed to be responsible for such changes in the LOD. Their periods have been used to infer the mean radial magnetic field strength in the core, a quantity otherwise inaccessible to observations (Gillet et al, 2010). In a sphere, the flow of these waves follows contours of constant column height. It has been proposed that such flows are also unable to exert any pressure torque in the non-axisymmetric case (Jault and Finlay, 2015). It is unknown how important domains without closed geostrophic contours are for the topographic torque. Such domains are certainly present in the Earth’s

State of the art on lower mantle electrical conductivity σ
From observed lack of reflected torsional oscillations: Only bounds are on conductance $G = \sigma H$, where H is the depth of the conducting region. Constraint is on $Q = \sqrt{\frac{\mu_0}{\rho}} G B_r\|_{z=0} \approx 10^{-5} G B_r\|_{z=0} \quad (SI)$ where $G = \int_{r_0}^{r_0+H} \sigma\, dr \approx \sigma H$. $Q \simeq 1$ is preferred.
Pros: This is the strongest constraint on conductance.
Cons: Need definitive bounds on reflection coefficient/reflected energy. Need bounds on radial field B_r at $z = 0$. Provides information on only one region of CMB, at the equator. Most of CMB entirely unconstrained. Conductance not required to be laterally homogeneous, could have isolated blobs. Theory for laterally heterogeneous conductance yet to be worked out.

Table 1.1: State of the art on lower mantle electrical conductivity σ at the base of the mantle.

core.

Understanding the influence of topography on the flow structure and periods of torsional waves is crucial to verify their robustness in predicting core quantities in any planetary or stellar core.

1.10 Conclusions

There have been spectacular achievements in core studies over the last decade. Not least is the observation of torsional oscillations. We have alluded to some of the open issues in preceding sections. Although much is understood, it has proven difficult to deduce concrete properties of the Earth. Tables 1 and 2 summarise the state of play on the most important issues.

State of the art on interior field strength
Gillet et al (2010,2015) provide a lower bound of 2-3mT in the cylindrically radial magnetic field strength. The profile of B_s shows weakening towards the CMB.
Pros: Almost exactly predicts the filtered length-of-day in the 5-8 year period range.
Cons: Has never been replicated.

Table 1.2: State of the art on interior field strength in the Earth's core.

Bibliography

Abarca Del Rio R, Gambis D, Salstein D (2000) Interannual signals in length of day and atmospheric angular momentum. In: Annales Geophysicae, Springer, vol 18, pp 347–364

Aubert J (2015) Geomagnetic forecasts driven by thermal wind dynamics in the Earth's core. Geophys J Int 203(3):1738–1751

Aubert J (2018) Geomagnetic acceleration and rapid hydromagnetic wave dynamics in advanced numerical simulations of the geodynamo. Geophys J Int 214(1):531–547

Aubert J, Finlay CC (2019) Geomagnetic jerks and rapid hydromagnetic waves focusing at Earth's core surface. Nature Geosci 12(5):393–398

Aubert J, Gillet N, Cardin P (2003) Quasigeostrophic models of convection in rotating spherical shells. Geophys Geochem Geosystems 4

Aubert J, Gastine T, Fournier A (2017) Spherical convective dynamos in the rapidly rotating asymptotic regime. J Fluid Mech 813:558–593

Aurnou J, King E (2017) The cross-over to magnetostrophic convection in planetary dynamo systems. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 473(2199):20160,731

Bärenzung J, Holschneider M, Wicht J, Sanchez S, Lesur V (2018) Modeling and predicting the short-term evolution of the geomagnetic field. Journal of Geophysical Research: Solid Earth 123(6):4539–4560

Barrois O, Gillet N, Aubert J (2017) Contributions to the geomagnetic secular variation from a reanalysis of core surface dynamics. Geophys J Int 211(1):50–68

Braginsky S (1970) Torsional magnetohydrodynamics vibrations in the earth's core and variations in day length. Geomagn Aeron 10:3–12

Buffett BA (1996a) Gravitational oscillations in the length of day. Geophysical Research Letters 23(17):2279–2282, DOI 10.1029/96GL02083

Buffett BA (1996b) A mechanism for decade fluctuations in the length of day. Geophysical Research Letters 23(25):3803–3806, DOI 10.1029/96GL03571

Buffett BA (2010) Chemical stratification at the top of earth's core: Constraints from observations of nutations. Earth and Planetary Science Letters 296(3-4):367–372

Buffett BA, Mathews PA, Herring TA (2002) Modelling of nutation and precession: Effects of electromagnetic coupling 107(B4):5.1–5.15

Calkins MA (2018) Quasi-geostrophic dynamo theory. Phys Earth Planet Int 276:182 – 189

Canet E, Fournier A, Jault D (2009) Forward and adjoint quasi-geostrophic models of the geomagnetic secular variation. J Geophys Res: Solid Earth 114(B11)

Chao B (2017) Dynamics of axial torsional libration under the mantle-inner core gravitational interaction. Journal of Geophysical Research: Solid Earth 122(1):560–571

Chao BF, Chung W, Shih Z, Hsieh Y (2014) Earth's rotation variations: a wavelet analysis. Terra Nova 26(4):260–264

Constable S (2007) Geomagnetism. Treatise on Geophysics, ed Schubert, G, Elsevier

Davies CJ, Stegman DR, Dumberry M (2014) The strength of gravitational core-mantle coupling. Geophysical Research Letters 41(11):3786–3792

Dormy E (2016) Strong-field spherical dynamos. J Fluid Mech 789:500–513

Finlay CC, Olsen N, Kotsiaros S, Gillet N, Tøffner-Clausen L (2016) Recent geomagnetic secular variation from Swarm. Earth, Planets and Space 68(1):1–18

Fournier A, Nerger L, Aubert J (2013) An ensemble Kalman filter for the time-dependent analysis of the geomagnetic field. Geochem geophys geosyst 14(10):4035–4043

Gastine T (2019) pizza: an open-source pseudo-spectral code for spherical quasi-geostrophic convection. Geophysical Journal International 217(3):1558–1576, DOI 10.1093/gji/ggz103, URL https://doi.org/10.1093/gji/ggz103, http://oup.prod.sis.lan/gji/article-pdf/217/3/1558/28225303/ggz103.pdf

Gastine T, Wicht J, Aubert J (2016) Scaling regimes in spherical shell rotating convection. Journal of Fluid Mechanics 808:690732, DOI 10.1017/jfm.2016.659

Gillet N (2019) Spatial And Temporal Changes Of The Geomagnetic Field: Insights From Forward And Inverse Core Field Models. In: Geomagnetism, aeronomy and space weather: a journey from the Earth's core to the sun, URL https://hal.archives-ouvertes.fr/hal-02042703

Gillet N, Jones CA (2006) The quasi-geostrophic model for rapidly rotating spherical convection outside the tangent cylinder. J Fluid Mech 554:343–369

Gillet N, Jault D, Canet E, Fournier A (2010) Fast torsional waves and strong magnetic field within the Earth's core. Nature 465(7294):74

Gillet N, Jault D, Finlay C (2015) Planetary gyre, time-dependent eddies, torsional waves, and equatorial jets at the earth's core surface. Journal of Geophysical Research: Solid Earth 120(6):3991–4013

Gillet N, Jault D, Canet E (2017) Excitation of travelling torsional normal modes in an earths core model. Geophysical Journal International 210(3):1503–1516

Gillet N, Huder L, Aubert J (2019) A reduced stochastic model of core surface dynamics based on geodynamo simulations. Geophys J Int 219(1):522–539

Guervilly C, Cardin P, Schaeffer N (2019) Turbulent convective length scale in planetary cores. Nature 570(7761):368–371

Hardy CM, Livermore PW, Niesen J, Luo J, Li K (2018) Determination of the instantaneous geostrophic flow within the three-dimensional magnetostrophic regime. Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 474(2218):20180,412

Holme R (1998) Electromagnetic core-mantle coupling ii: probing deep mantle conductance. The core-mantle boundary region 28:139–151

Holme R, De Viron O (2013) Characterization and implications of intradecadal variations in length of day. Nature 499(7457):202

Jault D, Finlay CC (2015) Waves in the core and mechanical core-mantle interactions. In: Schubert G, Olson P (eds) Treatise on Geophysics, Core Dynamics, 2nd edition, vol 8, Elsevier, Oxford, chap 8.09, pp 225–244

Kloss C, Finlay CC (2019) Time-dependent low-latitude core flow and geomagnetic field acceleration pulses. Geophysical Journal International 217(1):140–168

Kuang W, Chao BF (2001) Topographic core-mantle coupling in geodynamo modeling. Geophysical research letters 28(9):1871–1874

Labbé F, Jault D, Gillet N (2015) On magnetostrophic inertia-less waves in quasi-geostrophic models of planetary cores. Geophys Astrophys Fluid Dyn 109(6):587–610

Legaut G (2005) Ondes de torsion dans le noyau terrestre. PhD thesis

Lesur V, Wardinski I, Baerenzung J, Holschneider M (2017) On the frequency spectra of the core magnetic field Gauss coefficients. Phys Earth Planet Int

Li K, Jackson A, Livermore PW (2014) Variational data assimilation for a forced, inertia-free magnetohydrodynamic dynamo model. Geophysical Journal International 199(3):1662–1676

Maffei S (2016) Quasi-geostrophic models for fast dynamics in the earth's outer core. PhD thesis, ETH Zurich

More C, Dumberry M (2017) Convectively driven decadal zonal accelerations in Earth's fluid core. Geophysical Journal International 213(1):434–446

Mound JE, Buffett BA (2006) Detection of a gravitational oscillation in length-of-day. Earth and Planetary Science Letters 243(3-4):383–389

Roberts PH, Aurnou J (2011) On the theory of core-mantle coupling. Geophys Astrophys Fl Dyn 106(2):157–230

Sanchez S, Wicht J, Bärenzung J, Holschneider M (2019) Sequential assimilation of geomagnetic observations: perspectives for the reconstruction and prediction of core dynamics. Geophysical Journal International 217(2):1434–1450

Schaeffer N, Jault D (2016) Electrical conductivity of the lowermost mantle explains absorption of core torsional waves at the equator. Geophysical Research Letters 43(10):4922–4928

Schaeffer N, Jault D, Nataf HC, Fournier A (2017) Turbulent geodynamo simulations: a leap towards Earth's core. Geophys J Int 211(1):1–29

Schwaiger T, Gastine T, Aubert J (2019) Force balance in numerical geodynamo simulations: a systematic study. Geophys J Int 219(S1):S101–S114

Soloviev A, Chulliat A, Bogoutdinov S (2017) Detection of secular acceleration pulses from magnetic observatory data. Physics of the Earth and Planetary Interiors 270:128–142

Teed RJ, Jones CA, Tobias SM (2015) The transition to earth-like torsional oscillations in magnetoconvection simulations. Earth and Planetary Science Letters 419:22–31

Vidal J, Cbron D, ud Doula A, Alecian E (2019) Fossil field decay due to nonlinear tides in massive binaries. Astronomy & Astrophysics 629:A142, DOI 10.1051/0004-6361/201935658